Chapter 2: Measures of Dispersion

EXERCISE 2.1 [PAGES 26 - 27]

Exercise 2.1 | Q 1 | Page 26

Find range of the following data:
575, 609, 335, 280, 729, 544, 852, 427, 967, 250

SOLUTION

Here, largest value (L) = 967, smallest value (S) = 250
∴ Range = L − S
= 967 − 250
= 717

Exercise 2.1 | Q 2 | Page 26

The following data gives number of typing mistakes done by Radha during a week. Find the range of the data:

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
No. of mistake	15	20	21	12	17	10

SOLUTION

Here, largest value (L) = 21, smallest value (S) = 10
∴ Range = L − S = 21 − 10 = 11

Exercise 2.1 | Q 3 | Page 26

Find range of the following data.

Classes	62 – 64	64 – 66	66 – 68	68 – 70	70 – 72
Frequency	5	3	4	5	3

SOLUTION

Here, upper limit of the highest class (L) = 72,
lower limit of the lowest class (S) = 62
∴ Range = L − S
= 72 – 62
= 10

Exercise 2.1 | Q 4 | Page 26

Find the Q.D. for the following data.
3, 16, 8, 15, 19, 11, 5, 17, 9, 5, 3.

SOLUTION

The given data can be arranged in ascending order as follows:

3, 3, 5, 5, 8, 9, 11, 15, 16, 17, 19.

Here, n = 11

Q_1 = value of $\left(\frac{n+1}{4}\right)^{th}$ observation

= value of $\left(\frac{11+1}{4}\right)^{th}$ observation

= value of 3^{rd} observation

$\therefore Q_1 = 5$

Q_3 = value of $3\left(\frac{n+1}{4}\right)^{th}$ observation

= value of $3\left(\frac{11+1}{4}\right)^{th}$ observation

= value of $(3 \times 3)^{th}$ observation

= value of 9^{th} observation

$\therefore Q_3 = 16$

Q.D. = $\frac{Q_3 - Q_1}{2}$

= $\frac{16-5}{2}$

= $\frac{11}{2}$

$\therefore$ Q.D. = 5.5

Exercise 2.1 | Q 5 | Page 26

Given below are the prices of shares of a company for the last 10 days. Find Q.D.: 172, 164, 188, 214, 190, 237, 200, 195, 208, 230.

SOLUTION

The given data can be arranged in ascending order as follows:
164, 172, 188, 190, 195, 200, 208, 214, 230, 237
Here, n = 10

Q_1 = value of $\left(\frac{n+1}{4}\right)^{th}$ observation

= value of $\left(\frac{10+1}{4}\right)^{th}$ observation

= value of $(2.75)^{th}$ observation

= value of 2^{nd} observation + 0.75 (value of 3^{rd} observation – value of 2^{nd} observation)

= 172 + 0.75 (188 – 172)

= 172 + 0.75 (16)

= 172 + 12

= 184

Q_3 = value of $3\left(\frac{n+1}{4}\right)^{th}$ observation

= value of $3\left(\frac{10+1}{4}\right)^{th}$ observation

= value of $(3 \times 2.75)^{th}$ observation

= value of 8^{th} observation + 0.25 (value of 9^{th} observation – value of 8^{th} observation)

$= 214 + 0.25 (230 - 214)$

$= 214 + 0.25 (16)$

$= 214 + 4$

$= 218$

$$Q.D. = \frac{Q_3 - Q_1}{2}$$

$$= \frac{218 - 184}{2}$$

$$= \frac{34}{2}$$

$= 17$

Exercise 2.1 | Q 6 | Page 26

Calculate Q. D. for the following data:

X	24	25	26	27	28	29	30
F	6	5	3	2	4	7	3

SOLUTION

X	F	c.f. (less than type)
24	6	6
25	5	11 ← Q_1
26	3	14
27	2	16
28	4	20
29	7	27 ← Q_3
30	3	30

Total	**30**	

Here, N = 30

Q_1 = value of $\left(\frac{N+1}{4}\right)^{th}$ observation

= value of $\left(\frac{30+1}{4}\right)^{th}$ observation

= value of $(7.75)^{th}$ observation

Cumulative frequency which is just greater than (or equal to) 7.75 is 11.

$\therefore Q_1 = 25$

Q_3 = value of $\left[3\left(\frac{n+1}{4}\right)\right]^{th}$ observation

= value of $\left[3\left(\frac{30+1}{4}\right)\right]^{th}$ observation

= value of $(3 \times 7.75)^{th}$ observation

= value of $(23.25)^{th}$ observation

Cumulative frequency which is just greater than (or equal to) 23.25 is 27.

$\therefore Q_3 = 29$

$\therefore$ Q.D. = $\frac{Q_3 - Q_1}{2}$

= $\frac{29 - 25}{2}$

= $\frac{4}{2}$

$\therefore$ Q.D. = 2

Exercise 2.1 | Q 7 | Page 26

Following data gives the age distribution of 250 employees of a firm. Calculate Q.D. of

Age (in years)	20 – 25	25 – 30	30 – 35	35 – 40	40 – 45	45 – 50
No. of employees	30	40	60	50	46	14

the distribution:

SOLUTION

We construct the less than cumulative frequency table as follows:

Age (in years)	**No. of employees (f)**	**Less than cumulative frequency (c.f.)**
20 – 25	30	30
25 – 30	40	70 ← Q_1
30 – 35	60	130
35 – 40	50	180 ← Q_3
40 – 50	46	226
45 – 50	14	240
Total	**N = 240**	

Here, N = 240

For Q_1, class = class containing $\left(\frac{N}{4}\right)^{th}$ observation

$\therefore \frac{N}{4} = \frac{240}{4} = 60$

Cumulative frequency which is just greater than (or equal to) 60 is 70.

$\therefore Q_1$ Lies in the class 25 – 30.

$\therefore$ L = 25, f = 40, c.f. = 30, h = 5

$\therefore Q_1 = L + \frac{h}{f}\left(\frac{N}{4} - c.f.\right)$

$= 25 + \frac{5}{40}(60 - 30)$

$= 25 + \frac{1}{8}(30)$

= 25 + 3.75

$\therefore Q_1$ = 28.75

For Q_3 class = class containing $\left(\frac{3N}{4}\right)^{th}$ observation

$\therefore \frac{3N}{4} = \frac{3 \times 240}{4} = 180$

Cumulative frequency which is just greater than (or equal to) 180 is 180.

$\therefore$ Q_3 Lies in the class 40 – 50

$\therefore$ L = 35, f = 50, c.f. = 130, h = 5

$Q_3 = L + \frac{h}{f}\left(\frac{3N}{4} - c.f.\right)$

$= 35 + \frac{5}{50}(180 - 130)$

$= 35 + \frac{1}{10} \times 50$

= 35 + 5

$\therefore Q_3 = 40$

$\therefore$ Q.D. $= \frac{Q_3 - Q_1}{2}$

$= \frac{40 - 28.75}{2}$

$= \frac{11.25}{2}$

$\therefore$ Q.D. = 5.625

Exercise 2.1 | Q 8 | Page 27

Following data gives the weight of boxes.
Calculate Q.D. for the data:

Weight (kg.)	10 – 12	12 – 14	14 – 16	16 – 18	18 – 20	20 – 22
No. of boxes	3	7	16	14	18	2
c.f.	3	10	26	40	58	60

SOLUTION

Weight (kg.)	No. of boxes (f)	c.f.
10 – 12	3	3
12 – 14	7	10
14 – 16	16	26 ← Q_1
16 – 18	14	40
18 – 20	18	58 ← Q_3
20 – 22	2	60
Total	**N = 60**	

Here, N = 60

Q_1 class = class containing $\left(\frac{N}{4}\right)^{th}$ observation

$\therefore \frac{N}{4} = \frac{60}{4} = 15$

Cumulative frequency which is just greater than (or equal to) 15 is 26.

$\therefore$ Q_1 Lies in the class 14 – 16

$\therefore$ L = 14, f = 16, c.f. = 10, h = 2

$\therefore Q_1 = L + \frac{h}{f}\left(\frac{N}{4} - c.f.\right)$

$= 14 + \frac{2}{16}(15 - 10)$

$= 14 + \frac{1}{8} \times 5$

$= 14 + 0.625$

$\therefore Q_1 = 14.625$

Q_3 class = class containing $\left(\frac{3N}{4}\right)^{th}$ observation

$\therefore \frac{3N}{4} = \frac{3 \times 60}{4} = 45$

Cumulative frequency which is just greater than (or equal to) 45 is 58.

$\therefore Q_3$ Lies in the class 18 – 20

$\therefore L = 18, f = 18, c.f. = 40, h = 2$

$\therefore Q_3 = L + \frac{h}{f}\left(\frac{3N}{4} - c.f.\right)$

$= 18 + \frac{2}{18}(45 - 40)$

$= 18 + \frac{1}{9} \times 5$

$= 18 + 0.5556$

$\therefore Q_3 = 18.5556$

$\therefore Q.D. = \frac{Q_3 - Q_1}{2}$

$= \frac{18.5556 - 14.625}{2}$

$= \frac{3.9306}{2}$

$= 1.9653$

EXERCISE 2.2 [PAGES 30 - 31]

Exercise 2.2 | Q 1 | Page 30

Find the various and S.D. for the following set of numbers.
7, 11, 2, 4, 9, 6, 3, 7, 11, 2, 5, 8, 3, 6, 8, 8, 8, 2, 6

SOLUTION

Given data: 7, 11, 2, 4, 9, 6, 3, 7, 11, 2, 5, 8, 3, 6, 8, 8, 2, 6
The tabulated form of the above data is as given below:

x_1	2	3	4	5	6	7	8	9	11
f_1	3	2	1	1	3	2	3	1	2

We prepare the following table for the calculation of variance and S. D.

x_i	**f_i**	**f_ix_i**	**$f_ix_i^2$**
2	3	6	12
3	2	6	18
4	1	4	16
5	1	5	25
6	3	18	108
7	2	14	98
8	3	24	192
9	1	9	81
11	2	22	242
Total	N = 18	$\sum f_ix_i = 108$	$\sum f_ix_i^2 = 792$

$$\bar{x} = \frac{\sum f_i x_i}{N} = \frac{108}{18} = 6$$

$$\text{var}(X) = \sigma_x^2 = \frac{\sum f_i x_i^2}{N} - (\bar{x})^2$$

$$= \frac{792}{18} - (6)^2$$

$$= 44 - 36$$

$$= 8$$

$$\therefore \text{S.D. } \sigma_x = \sqrt{\text{Var}(X)}$$

$$= \sqrt{8}$$

$$= 2\sqrt{2}$$

Exercise 2.2 | Q 2 | Page 30

Find the various and S.D. for the following set of numbers.
65, 77, 81, 98, 100, 80, 129

SOLUTION

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
65	– 25	625
77	– 13	169
81	– 9	81
98	8	64
100	10	100
80	– 10	100
129	39	1521
$\sum x_i = 630$		$\sum (x_i - \bar{x})^2 = 2660$

Here, n = 7

$$\bar{x} = \frac{\sum x_i}{n} = \frac{630}{7} = 90$$

$$\text{Var (X)} = \sigma_x^2 = \frac{1}{n}\sum (x_i - \bar{x})^2 = \frac{2660}{7} = 380$$

$\therefore$ S.D.= σ_x

$= \sqrt{\text{Var}(X)}$

$= \sqrt{380}$

$= 2\sqrt{95}$

Exercise 2.2 | Q 3 | Page 31

Compute variance and standard deviation for the following data:

x	2	4	6	8	10
f	5	4	3	2	1

SOLUTION

We prepare the following table for the calculation of variance and S.D.:

x_i	f_i	f_ix_i	$f_ix_i^2$
2	5	10	20
4	4	16	64
6	3	18	108
8	2	16	128
10	1	10	100
Total	**N = 15**	**$\sum f_ix_i = 70$**	**$\sum f_ix_i^2 = 420$**

$$\bar{x} = \frac{\sum f_i x_i}{N} = \frac{70}{15} = 4.6667$$

$$\text{Var}(X) = \sigma_x^2 = \frac{\sum f_i x_i^2}{N} - (\bar{x})^2$$

$$= \frac{420}{15} - (4.6667)^2$$

$$= 28 - 21.7781$$

$$= 6.2219$$

$$\therefore \text{S.D.} = \sigma_x = \sqrt{\text{Var}(X)} = \sqrt{6.2219}$$

Exercise 2.2 | Q 4 | Page 31

Compute the variance and S.D.

x	1	3	5	7	9
Frequency	5	10	20	10	5

SOLUTION

We prepare the following table for the calculation of variance and S.D.:

x_i	f_i	f_ix_i	
1	5	5	
3	10	30	
5	20	100	
7	10	70	
9	5	45	
Total	**N = 50**	**$\sum f_ix_i = 250$**	

$$\bar{x} = \frac{\sum f_i x_i}{N} = \frac{250}{50} = 5$$

$$\text{Var}(X) = \sigma_x^2 = \frac{\sum f_i x_i^2}{N} - (\bar{x})^2$$

$$= \frac{1490}{50} - (5)^2$$

$$= 29.8 - 25$$

$$= 4.8$$

$$\therefore \text{S.D}, \sigma_x = \sqrt{\text{Var}(X)} = \sqrt{4.8}$$

Exercise 2.2 | Q 5 | Page 31

Following data gives age of 100 students in a school. Calculate variance and S.D.

Age (In years)	10	11	12	13	14
No. of students	10	20	40	20	10

SOLUTION

We prepare the following table for the calculation of variance and S.D:

Age (In years) x_i	**No. of students** f_i	f_ix_i	$f_ix_i^2$

10	10	100	1000
11	20	220	2420
12	40	480	5760
13	20	260	3380
14	10	140	1960
Total	**N = 100**	**$\sum f_i x_i = 1200$**	**$\sum f_i x_i^2 = 14520$**

$$\bar{x} = \frac{\sum f_i x_i}{N}$$

$$= \frac{1200}{100}$$

$$= 12$$

$$Var(X) = \sigma_x^2 = \frac{\sum f_i x_i^2}{N} - (\bar{x})^2$$

$$= \frac{14520}{100} - (12)^2$$

$$= 145.2 - 144$$

$$= 1.2$$

$$\therefore S.D. = \sigma_x = \sqrt{Var(X)} = \sqrt{1.2}$$

Exercise 2.2 | Q 6 | Page 31

The mean and variance of 5 observations are 3 and 2 respectively. If three of the five observations are 1, 3 and 5, find the values of other two observations.

SOLUTION

$\bar{x}$ = 3,Var (X) = 2, n = 5, x_1 = 1, x_2 = 3, x_3 = 5 ....[Given]

Let the remaining two observations be x_4 and x_5.

$$\bar{x} = \frac{\sum x_i}{n}$$

$\therefore \sum x_i = n\bar{x} = 5 \times 3 = 15$

$$Var(X) = \frac{\sum x_i^2}{n} - (\bar{x})^2$$

$$\therefore 2 = \frac{\sum x_i^2}{5} - (3)^2$$

$$\therefore \frac{\sum x_i^2}{5} = 2 + 9$$

$$\therefore \sum x_i^2 = 5 \times 11$$

$$\therefore \sum x_i^2 = 55$$

Now,

$\sum x_i = 1 + 3 + 5\ x_4 + x_5$

$\therefore 15 = 9 + x_4 + x_5$

$\therefore x_4 + x_5 = 15 - 9$

$\therefore x_4 + x_5 = 6$

$\therefore x_5 = 6 - x_4$ ....(i)

$\sum x_i^2 = 1^2 + 3^2 + 5^2 + x_4^2 + x_5^2$

$\therefore 55 = 1 + 9 + 25 + x_4^2 + (6 - x_4)^2$ .....[From (i)]

$\therefore 55 = 35 + x_4^2 + 36 - 12x_4 + \mathbf{x_4^2}$

$\therefore 2x_4^2 - 12x_4 + 16 = 0$

$\therefore x_4^2 - 6x_4 + 8 = 0$

$\therefore x_4^2 - 4x_4 - 2x_4 + 8 = 0$

$\therefore x_4(x_4 - 4) - 2(x_4 - 4) = 0$

$\therefore (x_4 - 4)(x_4 - 2) = 0$

$\therefore x_4 = 4$ or $x_4 = 2$

From (i), we get

$x_5 = 2$ or $x_5 = 4$

$\therefore$ The two numbers are 2 and 4.

Exercise 2.2 | Q 7 | Page 31

Obtain standard deviation for the following date:

Height (in inches)	60 – 62	62 – 64	64 – 66	66 – 68	68 – 70
Number of students	4	30	45	15	6

SOLUTION

We prepare the following table for the calculation of standard deviation.

Height (in inches)	Number of students f_i	Mid value x_i	f_ix_i	$f_ix_i^2$
60 – 62	4	61	244	14884
62 – 64	30	63	1890	119070
64 – 66	45	65	2925	190125
66 – 68	15	67	1005	67335
68 – 70	6	69	414	28566
Total	**N =100**		**$\sum f_ix_i$ = 6478**	**$\sum f_ix_i^2$ = 419980**

$$\bar{x} = \frac{\sum f_i x_i}{N} = \frac{6478}{100} = 64.78$$

$$\text{Var(X)} = \sigma_x^2 = \frac{\sum f_i x_i^2}{N} - (\bar{x})^2$$

$$= \frac{419980}{100} - (64.78)^2$$

$$= 4199.80 - 4196.4484$$

$$= 3.3516$$

$$\therefore \text{S.D.} = \sigma_x = \sqrt{\text{Var(X)}} = \sqrt{3.3516}$$

Exercise 2.2 | Q 8 | Page 31

The following distribution was obtained change of origin and scale of variable X.

d_i	– 4	– 3	– 2	– 1	0	1	2	3	4
f_i	4	8	14	18	20	14	10	6	6

If it is given that mean and variance are 59.5 and 413 respectively, determine actual class intervals.

SOLUTION

Here, Mean = $\bar{x}$ = 59.5 and

Var(X) = σ^2 = 413

d = $\frac{x-a}{h}$, where a is assumed mean and h is class width

We prepare the following table for calculation of the mean and variance of d_i

d_i	f_i	$f_i d_i$	$f_i d_i^2$
– 4	4	– 16	64
– 3	8	– 24	72
– 2	14	– 28	56
– 1	18	– 18	18
0	20	0	0
1	14	14	14
2	10	20	40
3	6	18	54
4	6	24	96
Total	N = 100	$\sum f_i d_i$ = – 10	$\sum f_i d_i^2$ = 414

$\bar{d} = \frac{1}{N}\sum f_i d_i$

$= \frac{1}{100} \times (-10) = -0.1$

Here, $\bar{d} = \frac{\bar{x} - a}{h}$

$\therefore -0.1 = \frac{59.5 - a}{h}$

$\therefore -0.1\ h = 59.5 - a$

$\therefore -0.1\ h + a = 59.5$ ....(i)

$\text{Var(D)} = \sigma_d^2 = \frac{1}{N}\sum f_i d_i^2 - (\bar{d})^2$

$= \frac{1}{100}(414) - (-0.1)^2$

$= 4.14 - 0.01$

$= 4.13$

Now, $\text{Var(X)} = h^2 \cdot \text{Var(D)}$

$\therefore 413 = h^2\ (4.13)$

$\therefore h^2 = \frac{413}{4.13}$

$\therefore h^2 = 100$

$\therefore h = 10$

Substituting h = 10 in equation (i), we get

$-0.1 \times 10 + a = 59.5$

$\therefore -1 + a = 59.5$

$\therefore 59.5 = a - 1$

$\therefore a = 59.5 + 1$

$\therefore a = 60.5$

We prepare the following table to determine actual class-intervals for corresponding values of d_i

d_i	Mid value $x_i = d_i \times h + a$	Class-interval
– 4	20.5	15.5 – 25.5
– 3	30.5	25.5 – 35.5
– 2	40.5	35.5 – 45.5
– 1	50.5	45.5 – 55.5
0	60.5	55.5 – 65.5
1	70.5	65.5 – 75.5
2	80.5	75.5 – 85.5
3	90.5	85.5 – 95.5
4	100.5	95.5 – 105.5

∴ The actual class intervals are 15.5 – 25.5, 25.5 – 35.5, …., 95.5 – 105.5

EXERCISE 2.3 [PAGES 33 - 34]

Exercise 2.3 | Q 1 | Page 33

Mean and standard deviation of two distributions of 100 and 150 items are 50, 5 and 40, 6 respectively. Find the mean and standard deviations of all the 250 items taken together.

SOLUTION

Here, $n_1 = 100$, $n_2 = 150$, $\bar{x}_1 = 50$, $\bar{x}_2 = 40$, $\sigma_1 = 5$, $\sigma_2 = 6$

Combined Mean is given by,

$$\bar{x}_c = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

$$= \frac{(100)(50) + (150)(40)}{100 + 150}$$

$$= \frac{5000 + 6000}{250}$$

$$= \frac{11000}{250}$$

= 44

Combined standard deviation is given by,

$$\sigma_c = \sqrt{\frac{n_1\left(\sigma_1^2 + d_1^2\right) + n_2\left(\sigma_2^2 + d_2^2\right)}{n_1 + n_2}}$$

where $d_1 = \bar{x}_1 - \bar{x}_c$, $d_2 = \bar{x}_2 - \bar{x}_c$

∴ $d_1 = 50 - 44 = 6$ and $d_2 = 40 - 44 = -4$

∴ $d_1 = 50 - 44 = 6$ and $d_2 = 40 - 44 = -4$

$$\therefore \sigma_c = \sqrt{\frac{100(5^2 + 6^2) + 150\left(6^2 + (-4)^2\right)}{100 + 150}}$$

$$= \sqrt{\frac{100(25 + 36) + 150(36 + 16)}{250}}$$

$$= \sqrt{\frac{100(61) + 150(52)}{250}}$$

$$= \sqrt{\frac{6100 + 7800}{250}}$$

$= \sqrt{\frac{13900}{250}}$

$= \sqrt{55.6}$

∴ The mean and standard deviation of all 250 items taken together are 44 and $\sqrt{55.6}$ respectively.

Exercise 2.3 | Q 2 | Page 33

For certain bivariate data, the following information is available.

	X	Y
Mean	13	17
S.D.	3	2
Size	10	10

Obtain the combined standard deviation.

SOLUTION

$\bar{x} = 13;\ \bar{y} = 17$, $\sigma_x = 3$; $\sigma_y = 2$, $n_x = 10$, $n_y = 10$.

Combined Mean,

$$\overline{x_c} = \frac{n_x\bar{x} + n_y\bar{y}}{n_x + n_y}$$

$$= \frac{10(13) + (10)(17)}{10 + 10}$$

$$= \frac{130 + 170}{20}$$

$$= \frac{300}{20}$$

∴ $\overline{x_c} = 15$

Combined standard deviation is given by,

$$\sigma_c = \sqrt{\frac{n_x(\sigma_x^2 + d_x^2) + n_y(\sigma_y^2 + d_y^2)}{n_x + n_y}}$$

Where, $d_1 = \bar{x} - \bar{x}_c, d_2 = \bar{y} - \bar{x}_c$

$\therefore$ d_1 = 13 – 15 = – 2 and d_2 = 17 –15 = 2.

$$\therefore \sigma_c = \sqrt{\frac{10\left[3^2 + (-2)^2\right] + 10(2^2 + 2^2)}{10 + 10}}$$

$$= \sqrt{\frac{10[9 + 4] + 10(4 + 4)}{20}}$$

$$= \sqrt{\frac{10(13) + 10(8)}{20}}$$

$$= \sqrt{\frac{130 + 80}{20}}$$

$$= \sqrt{\frac{210}{20}}$$

$$= \sqrt{10.5}$$

Exercise 2.3 | Q 3 | Page 33

Calculate coefficient of variation of marks secured by a student in the exam, where the marks are: 2, 4, 6, 8, 10.
(Given: $\sqrt{3.6}$ = 1.8974)

SOLUTION

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
2	−4	16
4	−2	4
6	0	0
8	2	4
10	4	16
$\sum x_i = 30$		$\sum (x_i - \bar{x})^2 = 40$

Here, n = 5

$\bar{x} = \frac{\sum x_i}{n}$

$= \frac{30}{5}$

$= 6$

$Var(X) = \sigma_x^2 = \frac{1}{n}\sum (x_i - \bar{x})^2 = \frac{40}{5} = 8$

$\therefore$ S.D. $= \sigma_x = \sqrt{Var(X)} = \sqrt{8} = 2.8284$

Now, C.V. $= \frac{\sigma}{\bar{x}} \times 100$

$= \frac{2.8284}{6} \times 100$

$= 47.14\%$

Exercise 2.3 | Q 4 | Page 33

Find the coefficient of Variation of a sample which has mean equal to 25 and standard deviation of 5.

SOLUTION

Given, $\bar{x}$ = 25, σ = 5

$$C.V. = \frac{\sigma}{x} \times 100$$

$$= \frac{5}{25} \times 100$$

= 20%

Exercise 2.3 | Q 5 | Page 33

A group of 65 students of class XI have their average height is 150.4 cm with coefficient of variation 2.5%. What is the standard deviation of their height?

SOLUTION

Given, n = 65, $\bar{x}$ = 150.4, C.V. = 2.5%

$$C.V. = \frac{\sigma}{\bar{x}} \times 100$$

$$\therefore 2.5 = \frac{\sigma}{150.4} \times 100$$

$$\therefore \frac{2.5 \times 150.4}{100} = \sigma$$

$$\therefore \sigma = \frac{376}{100}$$

$$\therefore \sigma = 3.76$$

∴ The standard deviation of students height is 3.76.

Exercise 2.3 | Q 6. (i) | Page 33

Two workers on the same job show the following results:

	Worker P	Worker Q
Mean time for completing the job (hours)	33	21
Standard Deviation (hours)	9	7

Regarding the time required to complete the job, which worker is more consistent?

SOLUTION

	Worker P	Worker Q
Mean time for completing the job (hours)	$\bar{p} = 33$	$\bar{q} = 21$
Standard Deviation (hours)	$\sigma_P = 9$	$\sigma_Q = 7$

C.V. (P) = $\frac{\sigma_P}{\bar{p}} \times 100$

$= \frac{9}{33} \times 100$

= 27.27%

C.V. (Q) = $\frac{\sigma_q}{\bar{q}} \times 100$

$= \frac{7}{21} \times 100$

= 33.33%

Since, C.V. (P) < C.V. (Q)

∴ Worker P is more consistent regarding the time required to complete the job.

Exercise 2.3 | Q 6. (ii) | Page 33

Two workers on the same job show the following results:

	Worker P	Worker Q
Mean time for completing the job (hours)	33	21
Standard Deviation (hours)	9	7

Which worker seems to be faster in completing the job?

SOLUTION

	Worker P	Worker Q
Mean time for completing the job (hours)	$\bar{p} = 33$	$\bar{q} = 21$
Standard Deviation (hours)	$\sigma_p = 9$	$\sigma_q = 7$

C.V. (P) = $\frac{\sigma_p}{\bar{p}} \times 100$

= $\frac{9}{33} \times 100$

= 27.27%

C.V. (Q) = $\frac{\sigma_q}{\bar{q}} \times 100$

= $\frac{7}{21} \times 100$

= 33.33%

Since, $\bar{p} > \bar{q}$

i.e., expected time for completing the job is less for worker Q.

∴ Worker Q seems to be faster in completing the job.

Exercise 2.3 | Q 7. (i) | Page 33

A company has two departments with 42 and 60 employees respectively. Their average weekly wages are Rs. 750 and Rs. 400. The standard deviations are 8 and 10 respectively. Which department has a larger bill?

SOLUTION

Let $n_1 = 42$, $n_2 = 60$, $\overline{x_1} = 750$, $\overline{x_2} = 400$, $\sigma_1 = 8$, $\sigma_2 = 10$

C.V. (1) = $100 \times \frac{\sigma_1}{\overline{x_1}} = 100 \times \frac{8}{750}$ = 1.07%

C.V. (2) = $100 \times \frac{\sigma_2}{\overline{x_2}} = 100 \times \frac{10}{400}$ = 2.5%

Since, $\overline{x_1} > \overline{x_2}$

i.e., average weekly wages are more for first department.

∴ first department has a larger bill.

Exercise 2.3 | Q 7. (ii) | Page 33

A company has two departments with 42 and 60 employees respectively. Their average weekly wages are Rs. 750 and Rs. 400. The standard deviations are 8 and 10 respectively. Which department has a larger variability in wages?

SOLUTION

Let $n_1 = 42, n_2 = 60, \overline{x_1} = 750, \overline{x_2} = 400, \sigma_1 = 8, \sigma_2 = 10$

$$C.V. (1) = 100 \times \frac{\sigma_1}{\overline{x_1}} = 100 \times \frac{8}{750} = 1.07\%$$

$$C.V. (2) = 100 \times \frac{\sigma_2}{\overline{x_2}} = 100 \times \frac{10}{400} = 2.5\%$$

Since, C.V. (1) < C.V. (2)

∴ second department is less consistent.

∴ second department has larger variability in wages.

Exercise 2.3 | Q 8 | Page 33

The following table gives weights of the students of class A. Calculate the Coefficient of variation.

$$\left(\text{Given} : \sqrt{0.8} = 0.8944\right)$$

Weight (in kg)	Class A
25 – 35	8
35 – 45	4
45 – 55	8

SOLUTION

Weight (in kg)	Class A (f_i)	Mid value (x_i)	f_ix_i	$f_ix_i^2$
25 – 35	8	30	240	7200
35 – 45	4	40	160	6400
45 – 55	8	50	400	20000
Total	**N = 20**		$\sum f_ix_i = 800$	$\sum f_ix_i^2 = 33600$

$\bar{x} = \frac{\sum f_i x_i}{N}$

$= \frac{800}{20}$

= 40

Var(X) = $\sigma_x^2 = \frac{\sum f_i x_i^2}{N} - (\bar{x})^2$

$= \frac{33600}{20} - (40)^2$

= 1680 − 1600

= 80

∴ S.D. = σ_x = $\sqrt{80}$

$= \sqrt{\frac{100 \times 8}{10}}$

$= 10\sqrt{0.8}$

= 10(0.8944)

= 8.944

C.V. (X) = $\frac{\sigma_x}{\bar{x}} \times 100$

$= \frac{8.944}{40} \times 100$

= 22.36%

Exercise 2.3 | Q 9 | Page 34

Compute coefficient of variation for team A and team B. $\left(\text{Given : } \sqrt{26} = 5.099, \sqrt{22} = 4.6904\right)$

No. of goals	0	1	2	3	4
No. of matches played by team A	18	7	5	16	14
No. of matches played by team B	14	16	5	18	17

Which team is more consistent?

SOLUTION

For team A

Let f_1 denote no. of goals of team A.

No. of goals (x_i)	No. of matches (f_{1i})	$f_{1i}x_i$	$f_{1i}x_i^2$
0	18	0	0
1	7	7	7
2	5	10	20
3	16	48	144
4	14	56	224
	$N_1 = 60$	$\sum f_{1i}x_i = 121$	$\sum f_{1i}x_i^2 = 395$

$$\overline{x_1} = \frac{\sum f_{1i}x_i}{N_1}$$

$$= \frac{121}{60}$$

= 2.0167

Standard deviation,

$$\sigma_{x_1}^2 = \frac{1}{N_1}\sum f_{1i}x_i^2 - (\bar{x}_1)^2$$

$$= \frac{395}{60} - (2.0167)^2$$

= 6.5833 − 4.0671

= 2.5162

$\therefore \sigma_{x1} = \sqrt{2.5162}$ = 1.5863

Co-efficient of variance;

$$\text{C.V}(x_1) = \frac{\sigma_{x1}}{\overline{x_1}} \times 100$$

$= \frac{1.5863}{2.0167} \times 100$

= 78.66%

For team B

Let f_2 denote no. of goals of team B.

No. of goals (x_i)	No. of matches (f_{2i})	$f_{2i}x_i$	$f_{2i}x_i^2$
0	14	0	0
1	16	16	16
2	5	10	20
3	18	544	162
4	17	68	272
	$N_2 = 70$	$\sum f_{2i}x_i = 148$	$\sum f_{2i}x_i^2 = 470$

$$\overline{x_2} = \frac{\sum f_{2i}x_i}{N_2}$$

$= \frac{148}{70}$

= 2.1143

Standard deviation,

$$\sigma_{x_2}^2 = \frac{1}{N_2}\sum f_{2i}x_i^2 - (\bar{x}_2)^2$$

$= \frac{470}{70} - (2.1143)^2$

= (6.7143 – 4.46703)

= 2.244

$\therefore \sigma_{x2} = \sqrt{2.244}$ = 1.4980

Co-efficient of variance;

$$\text{C.V.}(x_2) = \frac{\sigma_{x2}}{\overline{x_2}} \times 100$$

$$= \frac{1.4980}{2.1143} \times 100$$

$= 70.85\%$

Since, C.V. of team A > C.V. of team B.

∴ team B is more consistent.

Exercise 2.3 | Q 10 | Page 34

Given below is the information about marks obtained in Mathematics and Statistics by 100 students in a class. Which subject shows A the highest variability in marks?

	Mathematics	Statistics
Mean	20	25
S.D.	2	3

SOLUTION

Here, $\overline{x_m} = 20$, $\overline{x_s} = 25$, $\sigma_m = 2$, $\sigma_s = 3$

$$\text{C.V.}(M) = \frac{\sigma_m}{\overline{x_m}} \times 100$$

$$= \frac{2}{20} \times 100$$

$= 10\%$

$$\text{C.V.}(S) = \frac{\sigma_s}{\overline{x_s}} \times 100$$

$$= \frac{3}{25} \times 100$$

$= 12\%$

Since, C.V. (S) > C.V. (M)

∴ The subject statistics shows higher variability in marks.

MISCELLANEOUS EXERCISE 2 [PAGES 35 - 36]

Miscellaneous Exercise 2 | Q 1 | Page 35

Find the range for the following data.
116, 124, 164, 150, 149, 114, 195, 128, 138, 203, 144

SOLUTION

Here,
largest value (L) = 203,
smallest value (S) = 114
∴ Range = L – S
= 203 – 114
= 89.

Miscellaneous Exercise 2 | Q 2 | Page 35

Given below the frequency distribution of weekly wages of 400 workers. Find the range.

Weekly wages (in '00 ₹)	10	15	20	25	30	35	40
No. of workers	45	63	102	55	74	36	25

SOLUTION

Here,
largest value (L) = 40,
smallest value (S) = 10
∴ Range = L – S
= 40 – 10
= 30.

Miscellaneous Exercise 2 | Q 3 | Page 35

Find the range for the following data:

Classes	115 – 125	125 – 135	135 – 145	145 – 155	155 – 165	165 – 175
Frequency	1	4	6	1	3	5

SOLUTION

Here, upper limit of the highest class (L) = 175,
lower limit of the lowest class (S) = 115
∴ Range = L – S
= 175 – 115
= 60

Miscellaneous Exercise 2 | Q 4 | Page 35

The city traffic police issued challans for not observing the traffic rules:

Day of the Week	Mon	Tue	Wed	Thus	Fri	Sat
No. of Challans	40	24	36	58	62	80

Find Q.D.

SOLUTION

The given data can be arranged in ascending order as follows:
24, 36, 40, 58, 62, 80
Here, n = 6

Q_1 = value of $\left(\frac{n+1}{4}\right)^{th}$ observation

= value of $\left(\frac{6+1}{4}\right)^{th}$ observation

= value of $(1.75)^{th}$ observation

= value of 1^{st} observation + 0.75 (value of 2^{nd} observation – value of 1^{st} observation)

= 24 + 0.75(36 – 24)

= 24 + 0.75 (12)

= 24 + 9

$\therefore Q_1 = 33$

Q_3 = value of $3\left(\frac{n+1}{4}\right)^{th}$ observation

= value of $3\left(\frac{6+1}{4}\right)^{th}$ observation

= value of $(3 \times 1.75)^{th}$ observation

= value of $(5.25)^{th}$ observation

= value of 5^{th} observation + 0.25 (value of 6^{th} observation – value of 5^{th} observation)

= 62 + 0.25(80 – 62)

= 62 + 0.25(18)

= 62 + 4.5

= 66.5

$\therefore$ Q.D. = $\frac{Q_3 - Q_1}{2}$

= $\frac{66.5 - 33}{2}$

= $\frac{33.5}{2}$

= 16.75

Miscellaneous Exercise 2 | Q 5 | Page 35

Calculate Q.D. from the following data:

X (less than)	10	20	30	40	50	60	70
Frequency	5	8	15	20	30	33	35

SOLUTION

We construct the less than cumulative frequency table as follows:

Class	**f**	**Less than cumulative frequency (c.f.)**
0 – 10	5	5
10 – 20	3	8
20 – 30	7	15 ← Q_1
30 – 40	5	20
40 – 50	10	30 ← Q_3
50 – 60	3	33

60 – 70	2	35
Total	**N = 35**	

Here, N = 35

Q_1 class = class containing $\left(\frac{N}{4}\right)^{th}$ observation

$\therefore \frac{N}{4} = \frac{35}{4} = 8.75$

Cumulative frequency which is just greater than (or equal to) 8.75 is 15.

$\therefore Q_1$ lies in the class 20 – 30.

$\therefore$ L = 20, c.f. = 8, f = 7, h = 10

$$\therefore Q_1 = L + \frac{h}{f}\left(\frac{N}{4} - c.f.\right)$$

$$= 20 + \frac{10}{7}(8.75 - 8)$$

$$= 20 + \frac{10}{7} \times 0.75$$

$$= 20 + 1.07$$

$$\therefore Q_1 = 21.07$$

Q_3 class = class containing $\left(\frac{3N}{4}\right)^{th}$ observation

$\therefore \frac{3N}{4} = \frac{3 \times 35}{4}$ = 26.25

Cumulative frequency which is just greater than (or equal to) 26.25 is 30.

$\therefore$ Q_3 lies in the class 40 – 50.

$\therefore$ L = 40, c.f. = 20, f = 10, h = 10

$\therefore Q_3 = L + \frac{h}{f}\left(\frac{3N}{4} - c.f.\right)$

$= 40 + \frac{10}{10}(26.25 - 20)$

= 40 + 6.25

$\therefore$ Q_3 = 46.25

Q.D. = $\frac{Q_3 - Q_1}{2}$

$= \frac{46.25 - 21.07}{2}$

$= \frac{25.18}{2}$

= 12.59

Miscellaneous Exercise 2 | Q 6 | Page 35

Calculate the appropriate measure of dispersion for the following data:

Wages (In ₹)	Less than 35	35 – 40	40 – 45	45 – 50	50 – 55	55 – 60
No. of workers	15	50	85	40	27	33

SOLUTION

Since, open-ended classes are given, the appropriate measure of dispersion that we can compute is the quartile deviation.

We construct the less than cumulative frequency table as follows:

Wages (in ₹)	**No. of workers (f)**	**Less than cumulative frequency (c.f.)**

Less than 35	15	15
35 – 40	50	65 ← Q_1
40 – 45	85	150
45 – 50	40	190 ← Q_3
50 – 55	27	217
55 – 60	33	250
Total	**N = 250**	

Here, N = 250

Q_1 class = class containing $\left(\frac{N}{4}\right)^{th}$ observation

$\therefore \frac{N}{4} = \frac{250}{4} = 62.5$

Cumulative frequency which is just greater than (or equal to) 62.5 is 65.

$\therefore Q_1$ lies in the class 35 – 40.

$\therefore L = 35, c.f. = 15, f = 50, h = 5$

$\therefore Q_1 = L + \frac{h}{f}\left(\frac{N}{4} - c.f.\right)$

$= 35 + \frac{5}{50}(62.5 - 15)$

$= 35 + \frac{1}{10} \times 47.5$

$= 35 + 4.75$

$\therefore Q_1 = 39.75$

Q_3 class = class containing $\left(\frac{3N}{4}\right)^{th}$ observation

$\therefore \frac{3N}{4} = \frac{3 \times 250}{4} = 187.5$

Cumulative frequency which is just greater than (or equal to) 187.5 is 190.

$\therefore$ Q_3 lies in the class 45 – 50.

$\therefore$ L = 45, c.f. = 150, f = 40, h = 5

$\therefore Q_3 = L + \frac{h}{f}\left(\frac{3N}{4} - c.f.\right)$

$= 45 + \frac{5}{40}(187.5 - 150)$

$= 45 + \frac{1}{8} \times 37.5$

$= 45 + 4.6875$

$\therefore Q_3 = 49.6875$

$\therefore Q.D. = \frac{Q_3 - Q_1}{2}$

$= \frac{49.6875 - 39.75}{2}$

$= \frac{9.9375}{2}$

$\therefore$ Q.D. = 4.9688

Miscellaneous Exercise 2 | Q 7 | Page 35

Calculate Q.D. of the following data:

Height of plants (in feet)	2 – 4	4 – 6	6 – 8	8 – 10	10 – 12	14 – 14	14 – 16
No. of plants	15	20	25	12	18	13	17

SOLUTION

We construct the less than cumulative frequency table as follows:

Height of plants (in feet)	**No. of plants (f)**	**Less than cumulative frequency (c.f.)**

2 – 4	15	15
4 – 6	20	35 ← Q_1
6 – 8	25	60
8 – 10	12	72
10 – 12	18	90 ← Q_3
12 – 14	13	103
14 – 16	17	120
Total	**N = 120**	

Here, N = 120

Q_1 class = class containing $\left(\frac{N}{4}\right)^{th}$ observation

$\therefore \frac{N}{4} = \frac{120}{4} = 30$

Cumulative frequency which is just greater than (or equal to) 30 is 35.

$\therefore Q_1$ lies in the class 4 – 6

$\therefore L = 4$, c.f. = 15, f = 20, h = 2

$\therefore Q_1 = L + \frac{h}{f}\left(\frac{N}{4} - c.f.\right)$

$= 4 + \frac{2}{20}(30 - 15)$

$= 4 + \frac{1}{10} \times 15$

= 4 + 1.5

= 5.5

Q_3 class = class containing $\left(\frac{3N}{4}\right)^{th}$ observation

$\therefore \frac{3N}{4} = \frac{3 \times 120}{4} = 90$

Cumulative frequency which is just greater than (or equal to) 90 is 90.

$\therefore Q_3$ lies in the class 10 – 12

$\therefore L = 10$, c.f. = 72, f = 18, h = 2

$\therefore Q_3 = L + \frac{h}{f}\left(\frac{3N}{4} - c.f.\right)$

$= 10 + \frac{2}{18}(90 - 72)$

$= 10 + \frac{2}{18} \times 18$

= 10 + 2

$\therefore Q_3 = 12$

$\therefore$ Q.D. $= \frac{Q_3 - Q_1}{2}$

$= \frac{12 - 5.5}{2}$

$= \frac{6.5}{2}$

= 3.25

Miscellaneous Exercise 2 | Q 8 | Page 35

Find variance and S.D. for the following set of numbers.

25, 21, 23, 29, 27, 22, 28, 23, 27, 25

(Given $\sqrt{6.6} = 2.57$)

SOLUTION

We prepare the following table for the calculation of variance and S.D.:

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
25	0	0
21	– 4	16
23	– 2	4
29	4	16
27	2	4
22	– 3	9
28	3	9
23	– 2	4
27	2	4
25	0	0
$\sum x_i = 250$		$\sum (x_i - \bar{x})^2 = 66$

Here, n = 10

$$\bar{x} = \frac{\sum x_i}{n} = \frac{250}{10} = 25$$

$$\text{Var (X)} = \sigma_x^2 = \frac{1}{n}\sum (x_i - \bar{x})^2$$

$$= \frac{1}{10} \times 66$$

= 6.6

$\therefore$ S.D. = σ_x

$$= \sqrt{\text{Var (X)}}$$

$$= \sqrt{6.6}$$

= 2.57

Miscellaneous Exercise 2 | Q 9 | Page 35

Following data gives no. of goals scored by a team in 90 matches:

No. of goals scored	0	1	2	3	4	5
No. of matches	5	20	25	15	20	5

Compute the variance and standard deviation for the above data.

SOLUTION

We prepare the following table for the calculation of variance and S.D:

No. of goals scored (x_i)	No. of matches (f_i)	f_ix_i	$f_ix_i^2$
0	5	0	0
1	20	20	20
2	25	50	100
3	15	45	135
4	20	80	320
5	5	25	125
Total	N = 90	$\sum f_ix_i = 220$	$\sum f_ix_i^2 = 700$

$$\bar{x} = \frac{\sum f_ix_i}{N} = \frac{220}{90} = 2.44$$

$$\text{Var }(X) = \sigma_x^2$$

$$= \frac{\sum f_ix_i^2}{N} - (\bar{x})^2$$

$$= \frac{700}{90} - (2.44)^2$$

$$= 7.78 - 5.9536$$

$$= 1.83$$

S.D. = σ_x

$= \sqrt{\text{Var}(X)}$

$= \sqrt{1.83}$

Miscellaneous Exercise 2 | Q 10 | Page 35

Compute arithmetic mean and S.D. and C.V.

(Given $\sqrt{296}$ = 17.20)

C.I.	45 – 55	55 – 65	65 – 75	75 – 85	85 – 95	95 – 105
f	4	2	5	3	6	5

SOLUTION

We prepare the following table for the calculation of arithmetic mean and S.D.:

C.I.	**Mid value (xi)**	f_i	f_ix_i	$f_ix_i^2$
45 – 55	50	4	200	10000
55 – 65	60	2	120	7200
65 – 75	70	5	350	24500
75 – 85	80	3	240	19200
85 – 95	90	6	540	48600
95 – 105	100	5	500	50000
Total		N = 25	$\sum f_ix_i = 1950$	$\sum f_ix_i^2 = 159500$

Arthmetic mean = $\bar{x} = \frac{\sum f_i x_i}{N} = \frac{1950}{25}$ = 78

Var (X) = σ_x^2

= $\frac{\sum f_i x_i^2}{N} - (\bar{x})^2$

= $\frac{159500}{25} - (78)^2$

= 6380 – 6084

= 296

∴ S.D. = σ_x

= $\sqrt{\text{Var (X)}}$

= $\sqrt{296}$

= 17.20

C.V. = $100 \times \frac{\sigma_x}{\bar{x}}$

= $100 \times \frac{17.20}{78}$

= 22.05%

Miscellaneous Exercise 2 | Q 11 | Page 35

The mean and S.D. of 200 items are found to be 60 and 20 respectively. At the time of calculation, two items were wrongly taken as 3 and 67 instead of 13 and 17. Find the correct mean and variance.

SOLUTION

Here, n = 200, $\bar{x}$ = Mean = 60, S.D. = 20

Wrongly taken items are 3 and 67.

Correct items are 13 and 17.

Now, $\bar{x}$ = 60

$$\therefore \frac{1}{n}\sum_{i=1}^{n} x_i = 60$$

$$\therefore \frac{1}{200}\sum_{i=1}^{n} x_i = 60$$

$$\therefore \sum_{i=1}^{n} x_i = 200 \times 60$$

$$\therefore \sum_{i=1}^{n} x_i = 12000$$

Correct value of $\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i$ – (sum of wrongly taken items) + (sum of correct items)

= 12000 – (3 + 67) + (13 + 17)

= 12000 – 70 + 30

= 11960

Correct value of mean = $\frac{1}{n} \times \text{correct value of} \sum_{i=1}^{n} x_i = \frac{1}{200} \times 11960$ = 59.8

Now, S.D. = 20

Variance = $(S.D.)^2 = 20^2$

∴ Variance = 400

$$\therefore \frac{1}{n}\sum_{i=1}^{n} x_i^2 - (\bar{x})^2 = 400$$

$$\therefore \frac{1}{200}\sum_{i=1}^{n} x_i^2 - (60)^2 = 400$$

$$\therefore \frac{1}{200}\sum_{i=1}^{n} x_i^2 = 400 + 3600$$

$$\therefore \sum_{i=1}^{n} x_i^2 = 4000 \times 200$$

$$\therefore \sum_{i=1}^{n} x_i^2 = 800000$$

Correct value of $\sum_{i=1}^{n} x_i^2$

$= \sum_{i=1}^{n} x_i^2$ – (Sum of squares of wrongly taken items) + (Sum of squares of correct items)

$= 800000 - (3^2 + 67^2) + (13^2 + 17^2)$

$= 800000 - (9 + 4489) + (169 + 289)$

$= 800000 - 4498 + 458 = 795960$

∴ Correct value of Variance

$$= \left(\frac{1}{n} \times \text{correct value of } \sum_{i=1}^{n} x_i^2\right) - (\text{correct value of } \bar{x})^2$$

$= \frac{1}{200} \times 795960 - (59.8)^2$

= 3979.8 – 3576.04

= 403.76

∴ The correct mean is 59.8 and correct variance is 403.76.

Miscellaneous Exercise 2 | Q 12 | Page 35

The mean and S.D. of a group of 48 observations are 40 and 8 respectively. If two more observations 60 and 65 are added to the set, find the mean and S.D. of 50 items.

SOLUTION

n = 48, $\bar{x} = 40,\ \sigma_x$ = 8 ....[Given]

$\bar{x} = \frac{1}{n}\sum x_i$

∴ $\sum x_i = n\bar{x}$ = 48 × 40 = 1920

New $\sum x_i = \sum x_i + 60 + 65$

= 1920 + 60 + 65

= 2045

∴ New mean = $\frac{2045}{50}$ = 40.9

Now, σ_x = 8

∴ σ_x^2 = 64

Since, $\sigma_x^2 = \frac{1}{n}\left(\sum x_i^2\right) - (\bar{x})^2$

∴ 64 = $\frac{1}{48}\left(\sum x_i^2\right) - (40)^2$

∴ 64 = $\frac{1}{48}\left(\sum x_i^2\right) - 1600$

∴ $\frac{\sum x_i^2}{48}$ = 64 + 1600 = 1664

$\therefore \sum x_i^2 = 48 \times 1664 = 79872$

New $\sum x_i^2 = \sum x_i^2 + (60)^2\ (65)^2$

$= 79872 + 3600 + 4225$

$= 87697$

$\therefore$ New S.D. $= \sqrt{\frac{\text{New} \sum x_i^2}{n} - (\text{New mean})^2}$

$= \sqrt{\frac{87697}{50} - (40.9)^2}$

$= \sqrt{1753.94 - 1672.81}$

$= \sqrt{81.13}$

Miscellaneous Exercise 2 | Q 13 | Page 35

The mean height of 200 students is 65 inches. The mean heights of boys and girls are 70 inches and 62 inches respectively and the standard deviations are 8 and 10 respectively. Find the number of boys and combined S.D.

SOLUTION

Let n_1 and n_2 be the number of boys and girls respectively.

Let n = 200, $\overline{x_c}$ = 65, $\overline{x_1}$ = 70, $\overline{x_2}$ = 62, σ_1 = 8, σ_2 = 10

Here, $n_1 + n_2 = n$

$\therefore n_1 + n_2 = 200$ ....(i)

Combined mean is given by

$$\overline{x_c} = \frac{n_1\overline{x_1} + n_2\overline{x_2}}{n_1 + n_2}$$

$\therefore 65 = \dfrac{n_1(70) + n_2(62)}{200}$ ....[From (i)]

$\therefore 70n_1 + 62n_2 = 13000$

$\therefore 35n_1 + 31n_2 = 6500$ ....(ii)

Solving (i) and (ii), we get

$n_1 = 75, n_2 = 125$

Combined standard deviation is given by,

$$\sigma_c = \sqrt{\frac{n_1\left(\sigma_1^2 + d_1^2\right) + n_2\left(\sigma_2^2 + d_2^2\right)}{n_1 + n_2}}$$

where $d_1 = \overline{x_1} - \overline{x_c}$, $d_2 = \overline{x_2} - \overline{x_c}$

$\therefore d_1 = 70 - 65 = 5$ and $d_2 = 62 - 65 = -3$

$$\therefore \sigma_c = \sqrt{\frac{75(64 + 25) + 125(100 + 9)}{200}}$$

$$= \sqrt{\frac{6675 + 13625}{200}}$$

$$= \sqrt{\frac{20300}{200}}$$

$= \sqrt{101.5}$

= 10.07

Miscellaneous Exercise 2 | Q 14 | Page 35

From the following data available for 5 pairs of observations of two variables x and y, obtain the combined S.D. for all 10 observations.

where, $\sum_{i=1}^{n} x_i = 30$, $\sum_{i=1}^{n} y_i = 40$, $\sum_{i=1}^{n} x_i^2 = 225$, $\sum_{i=1}^{n} y_i^2 = 340$

SOLUTION

Here, $\sum_{i=1}^{n} x_i = 30$, $\sum_{i=1}^{n} y_i = 40$, $\sum_{i=1}^{n} x_i^2 = 225$, $\sum_{i=1}^{n} y_i^2 = 340$, $n_x = 5$, $n_y = 5$

$$\bar{x} = \frac{\sum x_i}{n_x} = \frac{30}{5} = 6,$$

$$\bar{y} = \frac{\sum y_i}{n_y} = \frac{40}{5} = 8$$

Combined mean is given by

$$\overline{x_c} = \frac{n_x\bar{x} + n_y\bar{y}}{n_x + n_y}$$

$$= \frac{5(6) + 5(8)}{5 + 5}$$

$$= \frac{30 + 40}{10}$$

$$= \frac{70}{10}$$

= 7

Combined standard deviation is given by,

$$\sigma_c = \sqrt{\frac{n_x\left(\sigma_x^2 + d_x^2\right) + n_y\left(\sigma_y^2 + d_y^2\right)}{n_x + n_y}}$$

Where $d_x = \bar{x} - \overline{x_c}$, $d_y = \bar{y} - \overline{x_c}$

$$\sigma_x^2 = \frac{1}{n_x}\sum x_i^2 - (\bar{x})^2$$

$$= \frac{1}{5}(225) - (6)^2$$

= 45 – 36

= 9

$$\sigma_y^2 = \frac{1}{n_y}\sum y_i^2 - (\bar{y})^2$$

$$= \frac{1}{5}(340) - (8)^2$$

= 68 – 64

= 4

$d_x = 6 - 7 = -1$ and $d_y = 8 - 7 = 1$

$$\therefore \sigma_c = \sqrt{\frac{5\left[9 + (-1)^2\right] + 5\left[4 + (1)^2\right]}{5 + 5}}$$

$$= \sqrt{\frac{5(9+1) + 5(4+1)}{10}}$$

$$= \sqrt{\frac{5(10) + 5(5)}{10}}$$

$$= \sqrt{\frac{50 + 25}{10}}$$

$$= \sqrt{\frac{75}{10}}$$

$$= \sqrt{7.5}$$

Miscellaneous Exercise 2 | Q 15 | Page 35

The mean and standard deviations of two brands of watches are given below:

	Brand-I	Brand-II
Mean	36 months	48 months
S.D.	8 months	10 months

Calculate a coefficient of variation of the two brands and interpret the results.

SOLUTION

Here, $\overline{x_I} = 36, \overline{x_{II}} = 48, \sigma_I = 8, \sigma_{II} = 10$

C.V. (I) = $100 \times \frac{\sigma_I}{\overline{x_I}} = 100 \times \frac{8}{36}$ = 22.22%

C.V. (II) = $100 \times \frac{\sigma_{II}}{\overline{x_{II}}} = 100 \times \frac{10}{48}$ = 20.83%

Since, C.V. (I) > C.V. (II)

∴ brand I is more variable.

Miscellaneous Exercise 2 | Q 16 | Page 36

Calculate coefficient of variation for the data given below [Given : $\sqrt{3.3} = 1.8166$]

C.I.	5 – 15	15 – 25	25 – 35	35 – 45	45 – 55	55 – 65	65 – 75
f	6	7	15	25	8	18	21

SOLUTION

C.I.	Mi value (x_i)	Frequency (f_i)	f_ix_i	$f_ix_i^2$
5 – 15	10	6	60	600
15 – 25	20	7	140	2800
25 – 35	30	15	450	13500
35 – 45	40	25	1000	40000
45 – 55	50	8	400	20000
55 – 65	60	18	1080	64800
65 – 75	70	21	1470	102900
Total		N = 100	$\sum f_ix_i$ = 4600	$\sum f_ix_i^2$ = 244600

$$\bar{x} = \frac{\sum f_ix_i}{N} = \frac{4600}{100} = 46$$

Var (X) = σ_x^2

$$= \frac{1}{N}\sum f_ix_i^2 - (\bar{x})^2$$

$$= \frac{1}{100} \times 244600 - (46)^2$$

= 2446 – 2116

= 330

S.D. = σ_x^2

$= \sqrt{330}$

$= \sqrt{3.3 \times 100}$

$= 10\sqrt{3.3}$

= 10 × 1.8166

= 18.166

$\therefore$ C.V. = $100 \times \frac{\sigma_x}{\bar{x}}$

= $100 \times \frac{18.166}{46}$

= 39.49%